Superconductivity meets topology: an edge supercurrent appears in a Weyl superconductor

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In the past decade, research on topological matter has spread to many corners in the field of quantum materials. As the wave of topological ideas expands, it has uncovered a deeper layer of physical reality and led to many predictions that have been confirmed. The focus to date has largely been on materials that are non-superconducting. In a recent experiment at Princeton, researchers have uncovered a new feature resulting from superconductivity in a topological material. The results were published in the May 1, 2020 issue of the journal *Science*.

Suppose we make a slip-knot on a shoelace and then glue its ends together. Unless we unglue the ends (or cut the shoelace), we cannot remove the knot. The winding implicit in the knot is a permanent property of the shoelace. Such topological arguments underlie many recent findings in topological materials.

Why are some materials topologically interesting (non-trivial) while others are topologically trivial? The basic idea may be explained as follows. In a metal, electrons occupy quantum states which can be regarded as rooms (with room number $k$) in a very large hotel. A permanent fixture in each room is an arrow $d$ that points along a direction in 3D space. The direction of $d$ varies smoothly from one room to the next. Your assignment is to record the direction of $d$ in each room by marking a spot on a toy globe with a magic marker pen. Starting at the penthouse, you mark its $d$ orientation, say at the north pole for specificity. As you work your way through all the rooms, the dense collection of marks produces a blot that gradually spreads over some fraction of the globe. An important feature is that if the pen “back tracks” to revisit a spot already marked, it erases that mark to leave the spot pristine.

On completion, if the last room (in the basement) has $d$ pointing to the south pole, the blot ends up covering the entire globe. Just like the knot in the shoelace, coating the globe is an action that cannot be undone. We say that the winding number is 1, and identify the hotel as topologically non-trivial.

By contrast, if $d$ in the last room ends up pointing back to the north pole, the initially advancing front of the blot must turn around and retreat back to the north pole. The shrinking of the blot leaves the globe uncovered. This identifies the hotel to be topologically trivial with winding number zero. (Technically, the hotel is called the Brillouin Zone of the lattice. The rooms are quantum states labelled by the momentum $k$. The vector $d$ is a compact representation of the Hamiltonian that describes the electronic properties specific to the material. The process of tracking $d$ on the globe is called the Pontryagin map, and the winding number is known as the Chern number.)

The hotel analogy is also helpful for explaining the chemical potential which we will need. To properly mimic a metal, the number of rooms must exceed the number of guests. As the hotel lacks elevators, we start at the ground floor and fill successively higher floors. The highest occupied floor is the analog of the chemical potential. As shown below, the properties of a metal or insulator reflect the behavior of electrons at the chemical potential (i.e. the occupants of the highest floor).

In the past decade, investigators have uncovered many topologically non-trivial materials. An important consequence of a non-zero Chern number in a topological material is the existence of surface (or edge) states. For a cubic crystal, the surface states spread over the six surfaces, while for an ultrathin square film the edge states spread over the four edges. In topological materials, the finite winding number ensures that the surface states are distinct from the ones in the bulk. (In the hotel analogy, they are rooms in a special wing of the hotel). They do not hybridize with the bulk states. In addition, when two branches
of surface states intersect, they ignore each other (by contrast in ordinary states the intersecting branches coalesce into “combinations” of each other). We say that the surface states are topologically protected.

The earliest examples of topological materials are the topological insulators. In the bulk, an energy gap opens up at the chemical potential. Gapping of the bulk states means that there are no states available near the chemical potential. However, the surface states remain ungapped because of topological protection. In the hotel analogy, the highest occupied floor is completely full and occupancy of the next 100 floors, say, is strictly forbidden (this corresponds to the energy gap). In the special wing, however, none of the floors are forbidden, so that additional guests can be accommodated.

In the topological insulator, the bulk is an insulator. In an applied electric field, none of the electrons can move into unoccupied states so the current is zero. In the hotel analogy, an electric field is equivalent to tilting the highest floor (as in the Titanic). Guests cannot tumble into neighboring rooms because all rooms are occupied (the available vacancies lie 100 floors above). Hence they stay put. In the special wing, however, the guests can freely tumble to unoccupied rooms; the collective tumbling is the analog of an electrical current that flows in response to the applied electric field. As a result, the surface states remain excellent conductors. Investigation of topological insulators in the past decade has uncovered a host of novel electronic phenomena.

In the Princeton experiment, the motivating question was what happens when the bulk is not an insulator but a superconductor. What novel features arise when superconductivity occurs in a topological material?

Superconductivity is a well-researched quantum phenomenon that occurs at low temperatures. When a superconductor (e.g. Pb, Al, Nb or cuprates) is cooled to a temperature below its critical temperature, the electrons pair up to form “Cooper pairs.” All Cooper pairs move to the same beat, so to speak. A rough analogy is a billion couples executing the same tightly-scripted dance choreography. The tight coordination contrasts with the random motion of electrons in the “normal state” that prevails at temperatures above the critical temperature.

The script that dictates how Cooper pairs behave is the superconductor’s wave function. As explained shortly, a slight twist of the wave function compels all Cooper pairs in a long wire to move with the same (superfluid) velocity to produce a supercurrent that flows without producing Joule heating. Superconductors are widely applied to produce intense magnetic fields in MRI machines.

To see how twisting the wave function leads to a supercurrent, we picture the wave function as a long ribbon that is tautly stretched along the length of the superconductor. For decorative purposes, ribbons are often given multiple twists before mounting on a wall. Similarly, the taut ribbon along the superconductor can host multiple twists. The superfluid velocity of the Cooper pairs is proportional to the pitch of the wave function (number of twists per unit length). If there are no twists along the entire ribbon, all Cooper pairs are stationary (no supercurrent flows). As we twist the ribbon, all pairs adopt the unique velocity dictated by the pitch, resulting in a supercurrent. The maximum pitch is reached when the supercurrent exceeds its critical value, which leads to destruction of the superconducting state.

If we expose the superconductor to a weak magnetic field, an additional contribution to the phase winding arises. The effect is most dramatic when we join the ends of the wire to form a closed loop. The amount of magnetic field enclosed by the loop is measured by the flux (technically the product of field strength and area enclosed). In the quantum world, the flux is actually made up of a universal quantity called the flux quantum. Roughly, we may compare the magnetic flux to a photograph in newsprint, in
which dark shades represent intense magnetic flux. Under magnification, the shading is revealed to be comprised of tiny dots (pixels). These are the analogs of flux quanta. The new feature is that each flux quantum entering the closed loop translates into an additional twist of the wave function ribbon. This flux-induced twisting adds to (or subtracts from) the twist already present due to the superfluid velocity.

In a closed loop, the two ends of the ribbon representing the wave function must be joined together. A little thought reveals that the total number of twists around the perimeter must be an exact integer. The closed loop cannot accept a non-integral number of twists. This suggests an apparent conflict between our classical world and the quantum world because, while we can continuously dial in an arbitrary value of the magnetic flux, the superconducting loop is unable to accommodate non-integral numbers of twists. The resolution is that there are two contributions to the number of twists, one flux-induced and the other tied to the superfluid velocity. The superconducting loop automatically adjusts the latter so that the two contributions add to the nearest integer. If the flux introduces say 3.2 twists, the superfluid velocity adjusts to “cancel” the extra 0.2 to result in exactly 3 twists. If the flux contribution increases to 3.7 twists, the superfluid velocity adjusts to add the extra 0.3 twist to bring the total to 4, and so forth. The result is that, as the magnetic flux increases smoothly, the compensating superfluid velocity varies with a saw-tooth profile to maintain the number of twists at integer values. This process is called fluxoid quantization.

In the experiment, Ong’s group investigated the critical current of the Weyl superconductor MoTe$_2$ cooled to a temperature (20 milliKelvin), well below its critical temperature 100 mK. When the applied magnetic flux is varied slowly, they observed an oscillation of the critical current that matches the behavior described above. To confirm that the oscillations arise from fluxoid quantization they verified that the oscillation frequency increases with the area of the crystal as predicted. The amplitude of the oscillations also matches the prediction of fluxoid quantization theory.

To the Princeton group’s knowledge, this is the first observation of an edge supercurrent in any superconductor. The reason why the edge condensate can remain independent of the bulk condensate is currently not well understood. From classical expectations, one would expect two fluid puddles (which is what the condensates are) that are in direct contact to merge into one. Yet the experiment shows that the edge condensate remains distinct from that in the bulk of the crystal. The research team speculates that the mechanism that keeps the two condensates from hybridizing is topological protection inherited from the protected edge states in MoTe$_2$. The group hopes to apply the same experimental technique to search for edge supercurrents in other unconventional superconductors. There are probably scores of them out there.

The study, "Evidence for an edge supercurrent in the Weyl superconductor MoTe2," by Wudi Wang, Stephan Kim, Minhao Liu, F. A. Cevallos, Robert. J. Cava and Nai Phuan Ong, was published in the journal Science on May 1, 2020. [https://doi.org/10.1126/science.aaw9270]